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THE CALCULATION OF THE ENERGY DEPOSITION IN SOFT TISSUE BY FAST NEUTRONS EMERGING FROM A NON-CAPTURING MEDIUM

An A. E. R. E. Report

BY

M.B. BIRAM

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THE CALCULATION OF THE ENERGY DEPOSITION IN SOFT TISSUE BY FAST NEUTRONS EMERGING FROM A NON - CAPTURING MEDIUM.

By

M. B. Biram

Reference A.E.R.E. T/R.443

ABSTRACT

The dose in rep. is calculated at interior points of a half space of soft tissue, for the case of neutrons entering the tissue from a non-capturing isotropically scattering medium. The variation of mean free path with energy is neglected, and, with this approximation, the biological effects can be shown to be proportional to the energy flux. This is then expressed as a function of depth of tissue using the Spherical Harmonics P_7 approximation.

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A.E.R.E., Harwell.
21st November, 1949.

1. Introduction.

The object of this report is to calculate the biological effect on the human body when monochromatic fast neutrons fall nearly isotropically on the surface of the tissue. The neutrons are supposed to travel through a non-capturing isotropically scattering medium in direct contact with the tissue. Consequently neutrons enter the tissue in nearly isotropic distribution.

The calculation is similar to that carried out in T/R 273 where the calculation of the energy deposition in soft tissue was made for neutrons entering at right angles to the free surface (the albedo case).

The conditions assumed in the present report correspond to a person leaning against the reflector of a pile and for convenience we shall refer to the non-capturing medium as the "reflector". For most practical considerations, such as neutrons emerging from the shield of a pile, neutrons will enter the body in a more anistropic manner than those coming from the reflector because the neutrons which have the greatest chance of penetrating the shield are those which have travelled the least distance in it, i.e. those emerging from the shield of the pile at right angles to it.

The calculations of the present report and those of T/R 273 cover the two extreme cases of normal incidence and almost isotropic incidence and they should be sufficient to assess the energy deposition in tissue for the intermediate cases.

The present calculations differ from those reported in T/R 273 in another detail. In the present case the body is in contact with a scattering medium so that the neutrons may be scattered successively between the body and the reflector. In T/R 273 the body was supposed to be effectively in vacuo. However the conditions of T/R 273 apply also if the body is in direct contact with a non-scattering absorbing medium and in this sense the two reports cover the two extreme cases of a pure absorber and a pure scatterer.

Successive scattering between the body and the reflector will of course, be negligible if the body is at a distance from the reflector which is large compared with the dimensions of the body. The present calculation is for most purposes pessimistic because it assumes direct contact.

The assumptions made in this analysis are the same as those in T/R 273.

The energy transferred to protons, which is a measure of the ionisation produced in a given volume element per unit time is given by

$$\frac{1}{2} \int E \frac{n(\underline{r}, v) \; vdv}{\iota p(v)}$$

where $\iota p(v)$ is the mean free path in tissue for collisions with a proton, $n(\underline{r}, v)dv$ is the density of neutrons with velocity v in the velocity interval dv, at the elementary volume whose position vector is $\underline{r}$ and E is the energy of the neutrons.

As in T/R 273, $\iota p(v)$ will be assumed constant. The over-estimations of neutron density and mean free path act in opposite directions in the integral and, to some extent, compensate each other.

It is true, also, that the neutrons which make the greatest contribution to the integral are those with highest energy. Hence the assumption of constant mean free path is justified. With this approximation, the integral is proportional to the neutron energy flux.

For simplification, the human body will be replaced by a half space of soft tissue, consisting of hydrogen, oxygen, nitrogen, carbon, sulpher and phosphorous, extending from $x = o$ to $x = +\infty$ where x is the depth of the tissue. It is assumed to be placed against the semi-infinite non-capturing isotropically scattering reflector, which extends from $x = -\infty$ to $x = o$. The source consists of neutrons entering the reflector at $x = -\infty$ and will be normalised so that the current is $1/\sqrt{3}$. It is possible to show that in this case the flux at a detector at $x = o$ in the absence of the tissue is unity. The reason for normalisation to unit flux rather than to unit current at $x = o$ is that flux is the quantity measured by a detector. Thus, this method allows of comparison of results for the isotropic source with those previously obtained for neutrons entering the tissue at right angles to it. The numerical calculations are carried out in this report for two values for the incident energy namely for 1 Mev and 5 Mev.

The law for the scattering of energy flux can be obtained by assuming that the nuclei of the atoms in the tissue, other than the hydrogen atoms, are infinitely heavy compared with the mass of a neutron. This law may be described fairly accurately as a fourth order polynominal in the cosine of the scattering angle. The angular distribution of energy flux can then be calculated using the Spherical Harmonics P_7 approximation. This process introduces four constants of integration for the solution in the tissue and four for the solution in the reflector. These can be obtained by equating the angular distribution in the tissue to that in the reflector at $x = o$

2. Equations for neutron energy flux.

It has been shown that the biological effect at a point is approximately proportional to the energy flux. Let this be denoted by $\varepsilon_o(x)$ where x is the depth of penetration of the neutron into the tissue and μ is the component of the velocity direction vector in the x direction.

Let us now consider the equation for the neutron energy flux in the tissue. This is given in T/R 273 where it has been shown that the angular distribution of neutron energy flux $\varepsilon(x,\mu)$ satisfies the equation:-

$$\mu \frac{\partial \varepsilon(x, \mu)}{\partial x} + \varepsilon(x, \mu) = \frac{1}{2\pi} \iint \varepsilon(x, \mu') \left[\frac{1-h}{2} + h(\mu_o^3 + |\mu_o|^3) \right] d\Omega' \qquad 1)$$

where a neutron travelling in direction $\underline{\Omega}'$ is scattered to a direction $\underline{\Omega}$, $\mu_c = \underline{\Omega} \cdot \underline{\Omega}'$, and $\varepsilon_o(x) = \int_{-1}^{+1} \varepsilon(x,\mu)\, d\mu$. h is the probability of collision with a proton, and 1-h the probability of collision with a heavy nucleus.

$$\text{Let} \quad \frac{1-h}{2} + h(\mu_o{}^2+|\mu_o|^2) = \Sigma \frac{2l+1}{2} g_l \; P_l(\mu)$$

$$\text{and} \quad \varepsilon(x,\mu) = \Sigma \frac{2l+1}{2} \varepsilon_l(x)P_l(\mu) \quad .$$

Substituting these expressions into 1) and equating coefficients of $P_l(\mu)$ on both sides of the equation we have

$$\dot{\varepsilon} = (g_0 - 1) \; \varepsilon_0$$

$$\text{and } (l+1)\dot{\varepsilon}_{l+1} + l\dot{\varepsilon}_{l-1} = (2l+1) \; \varepsilon_l \; (g - 1) \qquad \text{for} \quad l > o.$$

Since we are using a P_7 approximation, $\dot{\varepsilon}_8$ is neglected yielding the equations:-

$$\begin{aligned}
\dot{\varepsilon}_1 &= (g_0-1)\varepsilon_0 \\
2\dot{\varepsilon}_2 + \dot{\varepsilon}_0 &= 3(g_1-1)\varepsilon_1 \\
3\dot{\varepsilon}_3 + 2\dot{\varepsilon}_1 &= 5(g_2-1)\varepsilon_2 \\
4\dot{\varepsilon}_4 + 3\dot{\varepsilon}_2 &= 7(g_3-1)\varepsilon_3 \qquad\qquad 2) \\
5\dot{\varepsilon}_5 + 4\dot{\varepsilon}_3 &= 9(g_4-1)\varepsilon_4 \\
6\dot{\varepsilon}_6 + 5\dot{\varepsilon}_4 &= -11\varepsilon_5 \\
7\dot{\varepsilon}_7 + 6\dot{\varepsilon}_5 &= -13\varepsilon_6 \\
7\dot{\varepsilon}_6 &= -15\varepsilon_7
\end{aligned}$$

The solution of such a set of equations is discussed in detail in T/R 231 and much of the numerical work used for that report was incorporated in this one.

An expression is obtained for $\varepsilon_n(x)$ of the form

$$\varepsilon_n(x/l) = \sum_{s=1}^{4} a_n(\nu_s) \, e^{-\nu_s x/l} \qquad n, = o, 1, \ldots .7.$$

The ratios a_n/a_o and the values of the ν'_ss were obtained from previous calculations. The four a_o's are the constants of integration mentioned previously.

The equations for the energy flux in the reflector have now to be considered. In this medium the angular distribution satisfies the equation for isotropic scattering in one group theory.

$$\text{i.e.} \quad \mu \frac{\partial\varepsilon(x,\mu)}{\partial x} + \varepsilon(x,\mu) = \tfrac{1}{2} \; \varepsilon_0(x).$$

In the P_7 approximation $\varepsilon(x,\mu)$ is expanded as before

$$\varepsilon(x,\mu) = \sum_{l=0}^{7} \frac{2l+1}{2} \varepsilon_l(x) \; P_l(\mu) .$$

The $\varepsilon_l(x)$ in this case satisfy a set of equations similar to 2) with $g_0 = 1$ and $g_l = 0$ for $l \geq 1$

Hence it can be shown that

$$\varepsilon_0(x) = \sqrt{3}\,(A_1 - x/l) + \sum_{j=2}^{4} A_j \; e^{\nu_j x/l}$$

$$\varepsilon_1(x) = 1/\sqrt{3}$$

and for $7 \geq k \geq 2$ $\quad \varepsilon_k(x) = \sum_{j=2}^{4} A_j \; G_k(\nu_j) e^{\nu_j x/l}$

The formulae for the Gk's in terms of ν are given in MDDC 236 and may be calculated from there.

At the boundary $x = 0$, $\varepsilon(x,\mu)$ is continuous. Equating coefficients of the $P_l(\mu)$, eight equations are obtained in the eight unknown quantities $a_0(\nu_1)$, $a_0(\nu_2)$, $a_0(\nu_3)$, $a_0(\nu_4)$ A_1, A_2, A_3, A_4.

3. Numerical Results.

Two values of incident energy were considered, namely 1 Mev and 5 Mev. The values of h and of the mean free path for neutrons in soft tissue at these energies were given in T/R 273 and are as follows:-

E in Mev	h	l(mean free path) in cms
1	·775	2·82
5	·688	6·37

These values were then substituted in equations 2) and the energy flux $\varepsilon_0(x/l)$ determined at any point distance x. Now the energy absorbed per cc of tissue per second, for our normalisation is

$$\frac{E \quad \varepsilon_0(x/l)}{2lp}$$

To convert this into a dose, measured in rep. received during an eight hour period, this quantity is multiplied by a factor $10^{-4} \times 4.9494$.

This dose is plotted in Fig. I, as a function of distance into the tissue, for the cases of 1 Mev and 5 Mev. Also plotted in the same diagram are the corresponding curves for neutrons entering at right angles to the surface.

The angular distribution of emerging neutron energy flux for the case when neutrons come from the reflector is plotted in Fig. II.

As was expected, the surface dose was greater for the isotropic case than that when the neutrons enter at right angles to the surface, but the dose decreases more rapidly for greater depths in the tissue. It was found that the isotropic source gave increases of about 18% for both energy levels on the surface doses for the albedo case. The dose in each case is measured as the ratio of the actual dose to the flux measured at a detector in absence of the tissue.

The higher surface dose is due to the fact that neutrons entering at directions other than normal to the surface do not have to undergo so many collisions in order to be reflected back to the surface. Furthermore neutrons leaving the tissue can be reflected back into it.

It should be noted that the whole of the incident energy is absorbed in the present case, whereas in the albedo case a small fraction is reflected back. It will be remembered that the current is $1/\sqrt{3}$ while in the albedo case it is unity in the absence of the tissue. This explains why the area under the energy deposition/distance curve for the isotropic case is about $1/\sqrt{3}$ of that under the other.

Tolerance Flux of Fast Neutrons.

Assuming a high value, 20, for η the relative biological efficiency factor for protons, the maximum flux of fast neutrons from a reflector, measured at a detector in absence of the tissue, which in 8 hours gives a maximum biological effect of .1r of γ radiation is 55 for 1 Mev and 28 for 5 Mev.

An appreciable proportion of the surface dose is due to neutrons which have been reflected to and fro between the reflector and the tissue. If the tissue is placed in direct contact with the free surface along the whole of its length it will absorb all of these neutrons. As the body is moved away from the reflector, however, the chance of the tissue receiving neutrons which have been reflected in this way is diminished.

The past of the surface dose due to neutrons which are coming from the reflector after undergoing one or more reflections between it and the tissue is

$$\int_0^1 \varepsilon(o,\mu) \; d\mu - 1$$

This is about 10% of the whole surface dose.

Therefore if the body is at a distance from the reflector which is large compared with the dimensions of the body the surface dose will be about 10% smaller than indicated in Fig. I.

Acknowledgements.

The writer is indebted to Dr.J.H. Tait for suggesting the problem and method of solution and for his advice regarding various difficulties which arose.

References.

1) J.H. Tait "The application of the Weiner - Hopf and the Spherical Harmonics Methods to the case of energy deposition in a half-space consisting of pure hydrogen. T/R 231

2) J.H. Tait "The calculation of the tolerance flux of fast neutrons. T/R 273.

3) B. Carlson "Neutron Diffusion Spherical Harmonics Theory" MDDC 236 August 1946.

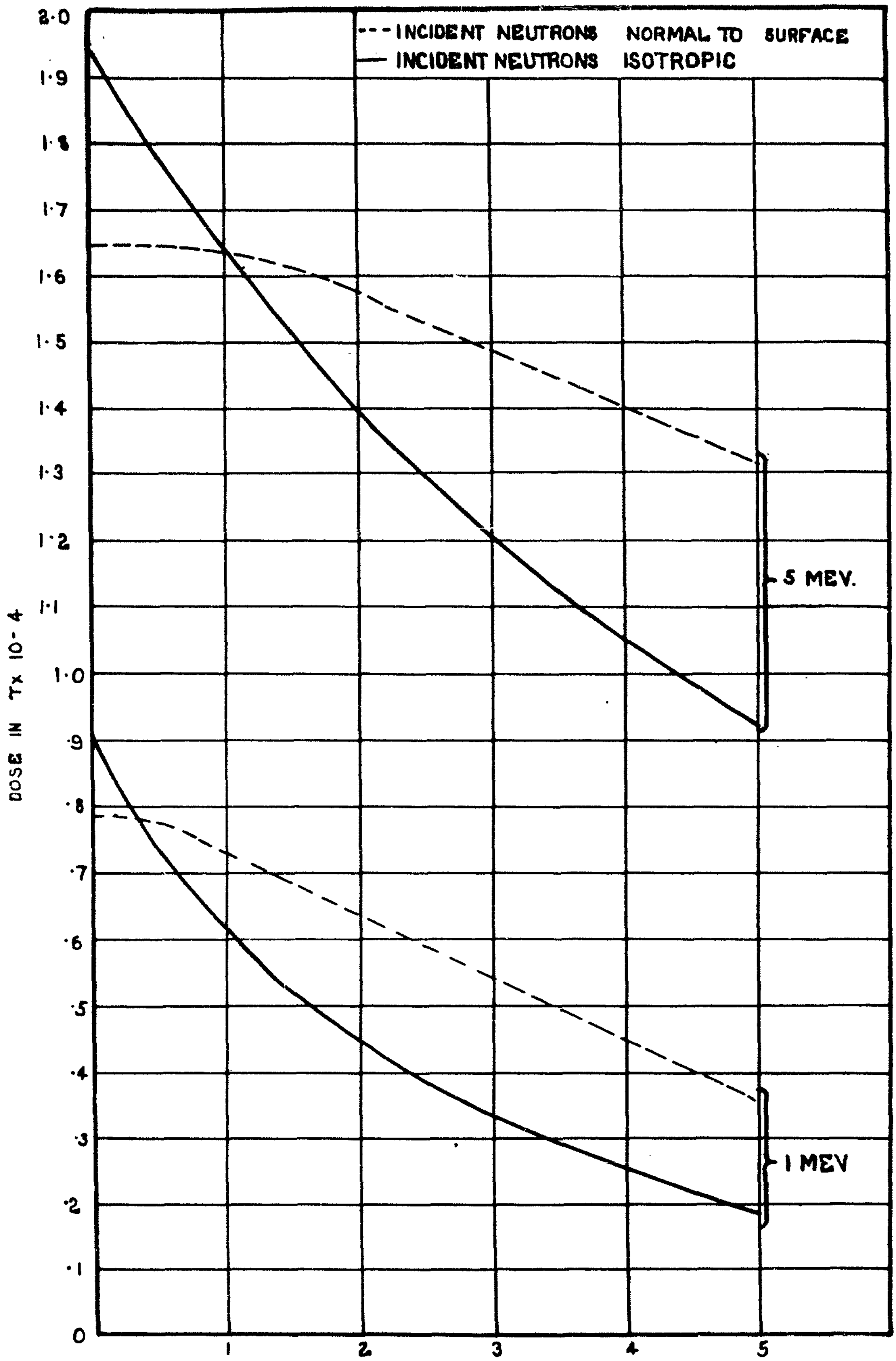

FIG 1. RATIO OF THE DOSE IN REP. RECEIVED IN THE TISSUE, DURING AN EIGHT HOUR PERIOD TO THE FLUX MEASURED AT A DETECTOR IN ABSENCE OF THE TISSUE.

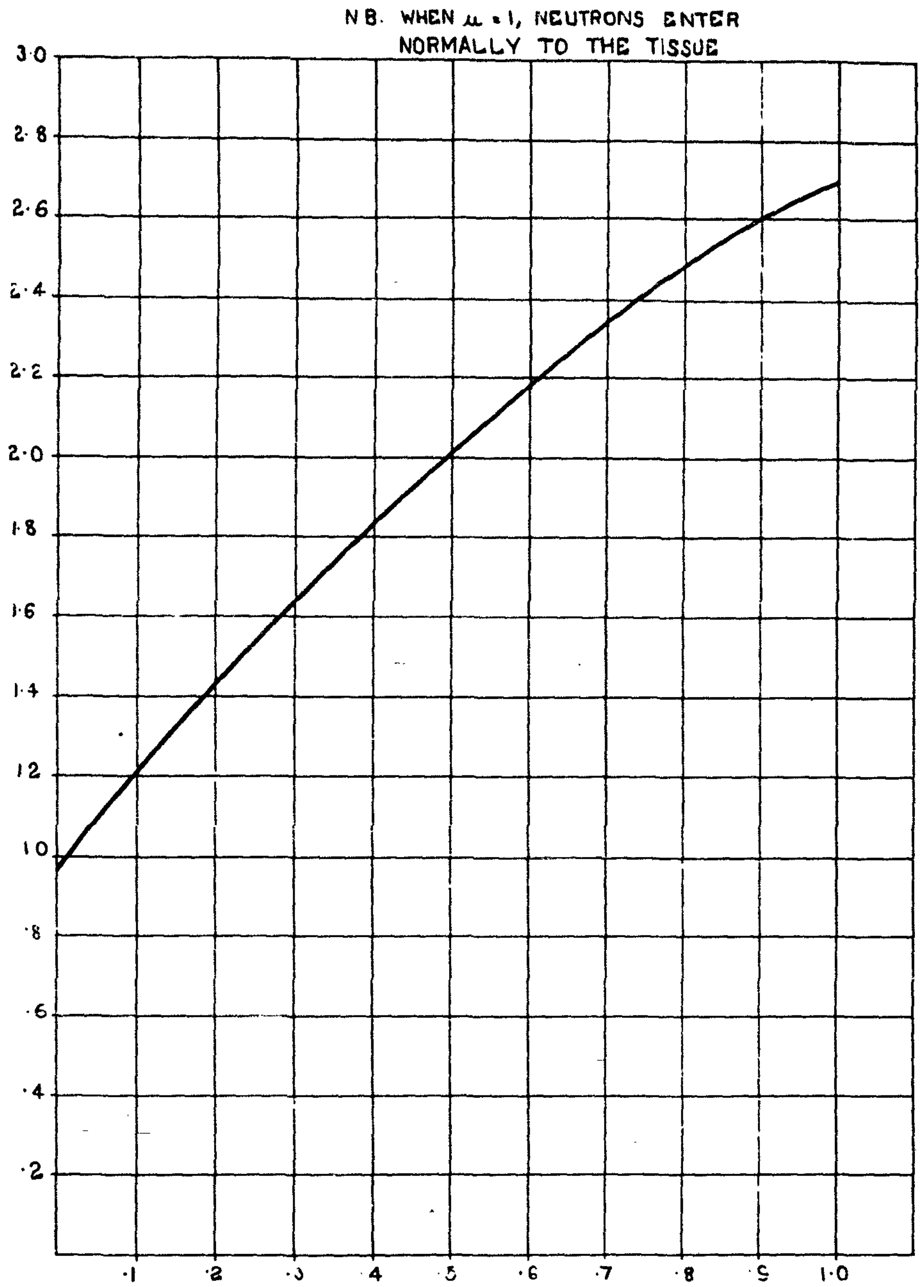

FIG. 2 ANGULAR DISTRIBUTION OF ENERGY FLUX AT THE SURFACE OF THE ISOTROPICALLY SCATTERING MEDIUM IN THE PRESENCE OF THE TISSUE.

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